Mark Scheme (Results)

Summer 2013

GCE Further Pure Mathematics 3 (6669/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes:

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. In some instances, the mark distributions (e.g. M1, B1 and A1) printed on the candidate's response may differ from the final mark scheme

General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x =

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x =

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c, \quad q \neq 0,$ leading to x =...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.
Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required.

Question Number	Scheme		Marks
	Mark (a) and (b) together		
1. (a) & (b)	$ae = 13$ **and** $a^2(e^2 - 1) = 25$	Sight of **both** of these (can be implied by their work) (allow ± ae = ±13 or ± ae = 13 or ae = ±13)	B1
	Solves to obtain $a^2 = ...$ or $a = ...$	Eliminates *e* to reach $a^2 = ...$ or $a = ...$	M1
	$a = 12$	Cao (not ± 12) unless -12 is rejected	A1
	e = 13/ "12"	Uses their *a* to find *e* or finds *e* by eliminating *a* (Ignore ± here) (Can be implied by a correct answer)	M1
	$x = (\pm)\frac{a}{e}, = \pm\frac{144}{13}$	M1: $(x =)(\pm)\frac{a}{e}$ $\pm$ not needed for this mark nor is *x* and even allow y = $(\pm)\frac{a}{e}$ here – just look for use of $\frac{a}{e}$ with numerical *a* and *e*. A1: $x = \pm\frac{144}{13}$ oe but must be an <u>equation</u> (Do not allow $x = \pm\frac{12}{13/12}$)	M1, A1
			Total 6
	If they use the eccentricity equation for the ellipse ($b^2=a^2(1-e^2)$) allow the M's		

Question Number	Scheme	Marks
2. (a)	$k\,\text{arsinh}\left(\frac{2x}{3}\right)\ (+c)$ or $k\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]\ (+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)\ (+c)$ or $\frac{1}{2}\ln[px+\sqrt{(p^2x^2+\frac{9}{4}p^2)}]\ (+c)$	A1
		(2)
(b)	So: $\frac{1}{2}\ln\left[6+\sqrt{45}\right]-\frac{1}{2}\ln\left[-6+\sqrt{45}\right]=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]$	M1
	Uses correct limits <u>and</u> combines logs	
	$=\frac{1}{2}\ln\left[\frac{6+\sqrt{45}}{-6+\sqrt{45}}\right]\left[\frac{6+\sqrt{45}}{6+\sqrt{45}}\right]=\frac{1}{2}\ln\left[\frac{(6+\sqrt{45})^2}{9}\right]$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\ln[2+\sqrt{5}]$ (or $\frac{1}{2}\ln[9+4\sqrt{5}]$)	A1cso
	Note that the last 3 marks can be scored without the need to rationalise e.g. $2\times\frac{1}{2}\left[\ln[2x+\sqrt{(4x^2+9)}]\right]_0^3=\ln(6+\sqrt{45})-\ln 3=\ln(\frac{6+\sqrt{45}}{3})$ M1: Uses the limits 0 and 3 and doubles M1: Combines Logs A1: $\ln[2+\sqrt{5}]$ oe	
		(3)
		Total 5
Alternative for **(a)**	$x=\frac{3}{2}\sinh u\Rightarrow\int\frac{1}{\sqrt{9\sinh^2 u+9}}.\frac{3}{2}\cosh u\,du=k\,\text{arsinh}\left(\frac{2x}{3}\right)\ (+c)$	M1
	$\frac{1}{2}\text{ar}\sinh\left(\frac{2x}{3}\right)\ (+c)$	A1
Alternative for **(b)**	$\left[\frac{1}{2}\text{arsinh}\left(\frac{2x}{3}\right)\right]_{-3}^{3}=\frac{1}{2}\text{arsinh}\ \ 2\ -\frac{1}{2}\text{arsinh}\ \ -2$	
	$\frac{1}{2}\ln(2+\sqrt{5})-\frac{1}{2}\ln(\sqrt{5}-2)=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2})$	M1
	Uses correct limits <u>and</u> combines logs	
	$=\frac{1}{2}\ln(\frac{2+\sqrt{5}}{\sqrt{5}-2}.\frac{\sqrt{5}+2}{\sqrt{5}+2})=\frac{1}{2}\ln(\frac{2\sqrt{5}+4+5+2\sqrt{5}}{5-4})$	M1
	Correct method to rationalise denominator (may be implied) **Method must be clear if answer does not follow their fraction**	
	$=\frac{1}{2}\ln[9+4\sqrt{5}]$	A1cso

Question Number	Scheme		Marks
3.	$(\frac{dx}{d\theta}) = 2\sinh 2\theta$ **and** $(\frac{dy}{d\theta}) = 4\cosh\theta$ Or equivalent correct derivatives		B1
	$A = (2\pi)\int 4\sinh\theta\sqrt{"2\sinh 2\theta"^2 + "4\cosh\theta"^2}\,d\theta$ or $A = (2\pi)\int 4\sinh\theta\sqrt{(1+(\frac{"4\cosh\theta"}{"2\sinh 2\theta"})^2}\,.2\sinh 2\theta\,d\theta$		M1
	Use of correct formula including replacing d*x* with $"2\sinh 2\theta"d\theta$ **if chain rule used. Allow the omission of the 2π here.**		
	$A = 32\pi\int \sinh\theta\cosh^2\theta\,d\theta$ $A = 32\pi\int(\sinh\theta + \sinh^3\theta)\,d\theta$		B1
	<u>Completely correct</u> expression for *A* with the square root removed This mark may be recovered later if the 2π is introduced later		
	A = $\frac{32\pi}{3}\left[\cosh^3\theta\right]_0^1$	M1: Valid attempt to integrate a correct expression or a multiple of a correct expression – dependent on the first M1 A1: Correct expression	dM1A1
	= $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M's A1: Cao and cso (no errors seen)	ddM1A1
			(7)

	Example Alternative Integration for last 4 marks		
	$\int\sinh\theta\cosh^2\theta\,d\theta = \int\sinh\theta(1+\sinh^2\theta)\,d\theta = \int(\sinh\theta+\sinh^3\theta)\,d\theta$ $\int(\sinh\theta + \frac{1}{4}\sinh 3\theta - \frac{3}{4}\sinh\theta)\,d\theta = \frac{1}{4}\int(\sinh\theta + \sinh 3\theta)\,d\theta$ $= \frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta$ **dM1:** $\int\sinh\theta\cosh^2\theta\,d\theta = p\cosh\theta + q\cosh 3\theta$ **A1**: $32\pi\left[\frac{1}{4}\cosh\theta + \frac{1}{12}\cosh 3\theta\right]$		dM1A1
	$A = 8\pi\left[\cosh\theta + \frac{1}{3}\cosh 3\theta\right]_0^1$ $= 8\pi(\cosh 1 + \frac{1}{3}\cosh 3 - \cosh 0 - \frac{1}{3}\cosh 0)$ .... $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	M1: Uses the limits 0 and 1 correctly. Dependent on **both** previous M's A1: Cao	ddM1A1

Question Number	Scheme		Marks
3.	**Alternative Cartesian Approach**		
	$x = 1 + \frac{y^2}{8}$	Any correct Cartesian equation	B1
	$\frac{dx}{dy} = \frac{y}{4}$ or $\frac{dy}{dx} = \frac{\sqrt{2}}{(x-1)^{\frac{1}{2}}}$	Correct Derivative	B1
	$A = \int 2\pi . y\sqrt{(1+\left(\frac{y}{4}\right)^2})dy$ or $A = \int 2\pi . \sqrt{8}(x-1)^{\frac{1}{2}}\sqrt{(1+\left(\frac{2}{x-1}\right))}dx$		M1
	Use of a correct formula		
	$A = 2\pi \times \frac{2}{3} \times 8\left(1+\frac{y^2}{16}\right)^{\frac{3}{2}}$ or $A = \frac{4\pi\sqrt{8}}{3}\ \ x+1\ ^{\frac{3}{2}}$		dM1 A1
	M1: Convincing attempt to integrate a relevant expression – dependent on the first M1 but allow the omission of 2π		
	A1: Completely correct expression for A		
	$A = 2\pi \times \frac{2}{3} \times 8\ \ 1+\sinh^2 1\ ^{\frac{3}{2}} - 2\pi \times \frac{2}{3} \times 8$ or $2\pi \times \frac{2}{3} \times \sqrt{8}\ \ 1+\cosh 2\ ^{\frac{3}{2}} - \frac{32\pi}{3}$		ddM1
	Correct use of limits (0 → 4sinh1 for y or 1 → cosh2 for *x*)		
	Use $1+\sinh^2 1 = \cosh^2 1$ to give $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	Use $\cosh 2 = 2\cosh^2 1 - 1$ to give $\frac{32\pi}{3}\left[\cosh^3 1 - 1\right]$	A1

Question Number	Scheme		Marks
4.	$\frac{dy}{dx} = \frac{40}{\sqrt{(x^2-1)}}$ - 9	M1: $\frac{dy}{dx} = \frac{p}{\sqrt{(x^2-1)}} - q$ A1: Cao	M1 A1
	Put $\frac{dy}{dx}$ **= 0** and obtain $x^2 = ...$ **(Allow sign errors only)**	e.g. $\left(\frac{1681}{81}\right)$	dM1
	$x = \frac{41}{9}$	M1: Square root A1: $x = \frac{41}{9}$ or exact equivalent (not $\pm\frac{41}{9}$)	M1 A1
	$y = 40\ln\{(\frac{41}{9}) + \sqrt{(\frac{41}{9})^2 - 1}\} - "41"$	Substitutes $x = "\frac{41}{9}"$ into the curve and uses the logarithmic form of arcosh	M1
	So y = 80ln 3 - 41	Cao	A1
			Total 7

Question Number	Scheme		Marks
5. (a) (i)&(ii)	$\begin{pmatrix}1&1&a\\2&b&c\\-1&0&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}1+a\\b+c\\1\end{pmatrix}=\lambda_1\begin{pmatrix}0\\1\\1\end{pmatrix}$, and so a = -1, λ_1 = 1		M1, A1, A1
	M1: Multiplies out matrix with first eigenvector and puts equal to λ_1 times eigenvector. A1 : Deduces a = -1. A1: Deduces λ_1 = 1		
	$\begin{pmatrix}1&1&a\\2&b&c\\-1&0&1\end{pmatrix}\begin{pmatrix}1\\0\\-1\end{pmatrix}=\begin{pmatrix}1-a\\2-c\\-2\end{pmatrix}=\lambda_2\begin{pmatrix}1\\0\\-1\end{pmatrix}$, and so $c = 2$, $\lambda_2 = 2$		M1, A1, A1
	M1: Multiplies out matrix with second eigenvector and puts equal to λ_2 times eigenvector. A1: Deduces c = 2. A1: Deduces λ_2 = 2		
	$b + c = \lambda_1$ so b = -1	M1: Uses $b + c = \lambda_1$ with their λ_1 to find a value for b (They must have an equation in b and c from the first eigenvector to score this mark) A1: b = -1	M1A1
	(a = -1, b = -1, c = 2, λ_1=1, λ_2 = 2)		(8)
(b)(i)	detP = -d - 1	Allow 1 – d – 2 or 1 – (2 + d) A correct (possibly un-simplified) determinant	B1
(ii)	$\mathbf{P}^T=\begin{pmatrix}1&2&-1\\1&1&0\\0&d&1\end{pmatrix}$ **or** minors $\begin{pmatrix}1&d+2&1\\1&1&1\\d&d&-1\end{pmatrix}$ **or** cofactors $\begin{pmatrix}1&-2-d&1\\-1&1&-1\\d&-d&-1\end{pmatrix}$ a **correct** first step		B1
	$\dfrac{1}{-d-1}\begin{pmatrix}1&-1&d\\-2-d&1&-d\\1&-1&-1\end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1 A1 A1
			(5)
			Total 13

Question Number	Scheme		Marks
6(a)	$I_n = \int_0^4 \underline{x^{n-1}\times x(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{\frac{1}{2}}$ prior to integration A1: Correct underlined expression (can be implied by their integration)	M1A1
	$I_n = \left[-\frac{1}{3}x^{n-1}(16-x^2)^{\frac{3}{2}}\right]_0^4 + \frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)^{\frac{3}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$\therefore I_n = \frac{n-1}{3}\int_0^4 x^{n-2}(16-x^2)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		
	i.e. $I_n = \frac{16(n-1)}{3}I_{n-2} - \frac{n-1}{3}I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+\frac{n-1}{3}) = \frac{16(n-1)}{3}I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*cso
			(6)
Way 2	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x = \int_0^4 x^n\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x = \int_0^4\frac{16x^n}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x - \int_0^4\frac{x^{n+2}}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$		
	$= \int_0^4 16x^{n-1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x - \int_0^4 x^{n+1}\times x(16-x^2)^{-\frac{1}{2}}\mathrm{d}x$		M1A1
	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expressions		
	$=\left[-16x^{n-1}(16-x^2)^{\frac{1}{2}}\right]_0^4 + 16(n-1)\int_0^4 x^{n-2}(16-x^2)^{\frac{1}{2}}\mathrm{d}x$ $-(\left[-x^{n+1}(16-x^2)^{\frac{1}{2}}\right]_0^4 + (n+1)\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x)$		dM1
	dM1: Parts in the correct direction on both (Ignore limits)		
	$I_n = 16(n-1)I_{n-2} - (n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1) = 16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*
Way 3	$\int_0^4 x^n(16-x^2)^{\frac{1}{2}}\mathrm{d}x = \int_0^4 x\times x^{n-1}\frac{(16-x^2)}{(16-x^2)^{\frac{1}{2}}}\mathrm{d}x$	M1: Obtains $x(16-x^2)^{-\frac{1}{2}}$ prior to integration A1: Correct expression	M1A1
	$=\left[-x^{n-1}(16-x^2)(16-x^2)^{\frac{1}{2}}\right]_0^4 + \int_0^4(16(n-1)x^{n-2} - (n+1)x^n)(16-x^2)^{\frac{1}{2}}\mathrm{d}x$		dM1
	dM1: Parts in the correct direction (Ignore limits)		
	$I_n = 16(n-1)I_{n-2} - (n+1)I_n$	Manipulates to obtain at least one integral in terms of I_n or I_{n-2} on the rhs.	M1
	$I_n(1+n+1) = 16(n-1)I_{n-2}$	Collects terms in I_n from both sides	M1
	$(n+2)I_n = 16(n-1)I_{n-2}$ *	Printed answer with no errors	A1*

Question Number	Scheme		Marks
(b)	$I_1 = \int_0^4 x\sqrt{(16-x^2)}\,dx = \left[-\frac{1}{3}(16-x^2)^{\frac{3}{2}}\right]_0^4 = \frac{64}{3}$	M1: Correct integration to find I_1 A1: $\frac{64}{3}$ or equivalent (May be implied by a later work – they are not asked explicitly for I_1)	M1 A1
	$\frac{64}{3}$ must come from correct work		
	Using $x = 4\sin\theta$: $I_1 = \int_0^{\frac{\pi}{2}} 4\sin\theta\sqrt{(16-16\sin^2\theta)}\,4\cos\theta\,d\theta = \int_0^{\frac{\pi}{2}} 64\sin\theta\cos^2\theta\,d\theta$ $= \left[-\frac{64}{3}\cos^3\theta\right]_0^{\frac{\pi}{2}}$ M1: A <u>complete</u> substitution and attempt to substitute <u>changed</u> limits A1: $\frac{64}{3}$ or equivalent		
	$I_5 = \frac{64}{7}I_3,\ I_3 = \frac{32}{5}I_1$	Applies to apply reduction formula twice. First M1 for I_5 in terms of I_3, second M1 for I_3 in terms of I_1 (Can be implied)	M1, M1
	$I_5 = \frac{131072}{105}$	Any <u>**exact**</u> equivalent (Depends on all previous marks having been scored)	A1
			(5)
			Total 11

Question Number	Scheme		Marks
7(a)	$\left(\frac{dx}{d\theta}=-a\sin\theta\right.$ and $\frac{dy}{d\theta}=b\cos\theta\Big)$ so $\frac{dy}{dx}=\frac{b\cos\theta}{-a\sin\theta}$		M1 A1
	M1: Differentiates both x and y and divides correctly A1: Fully correct derivative		
	Alternative: M1: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\Rightarrow\frac{2x}{a^2}+\frac{2yy'}{b^2}=0\Rightarrow y'=-\frac{b^2x}{a^2y}=-\frac{b^2a\cos\theta}{a^2b\sin\theta}$ Differentiates implicitly **and** substitutes for x and y A1: $=-\frac{b\cos\theta}{a\sin\theta}$		
	Normal has gradient $\frac{a\sin\theta}{b\cos\theta}$ or $\frac{a^2y}{b^2x}$	Correct perpendicular gradient rule	M1
	$(y-b\sin\theta)=\frac{a\sin\theta}{b\cos\theta}(x-a\cos\theta)$	Correct straight line method using a 'changed' gradient which is a function of θ	M1
	If $y = mx + c$ is used need to find c for M1		
	$ax\sin\theta-by\cos\theta=(a^2-b^2)\sin\theta\cos\theta$ *		A1
	Fully correct completion to printed answer		
			(5)
(b)	$x=\frac{(a^2-b^2)\cos\theta}{a}$	Allow un-simplified	B1
	$y=-\frac{(a^2-b^2)\sin\theta}{b}$	Allow un-simplified	B1
	$\left(=\frac{1}{2}\frac{(a^2-b^2)^2\cos\theta\sin\theta}{ab}\right)$	$=\frac{1}{4}\frac{(a^2-b^2)^2}{ab}\sin 2\theta$	M1A1
	M1: Area of triangle is $\frac{1}{2}$"OA"$\times$"OB" and uses double angle formula correctly **A1: Correct expression for the area (must be positive)**		
			(4)
(c)	Maximum area when $\sin 2\theta=1$ so $\theta=\frac{\pi}{4}$ or 45	Correct value for θ (may be implied by correct coordinates)	B1
	So the point P is at $\left(\frac{a}{\sqrt{2}},\frac{b}{\sqrt{2}}\right)$ oe $\left(a\cos\frac{\pi}{4},\ b\sin\frac{\pi}{4}\right)$ scores B1M1A0	M1: Substitutes their value of θ where $0<\theta<\frac{\pi}{2}$ or $0<\theta<90$ into their parametric coordinates A1: Correct exact coordinates	M1 A1
	Mark part (c) independently		
			(3)
			Total 12

Question Number	Scheme		Marks
8(a)	$(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$	Attempt scalar product	M1
	$\left\lvert\dfrac{(\mathbf{6i}+\mathbf{2j}+\mathbf{12k}).(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})-5}{\sqrt{3^2+4^2+2^2}}\right\rvert$	Use of correct formula	M1
	$\sqrt{29}$ (not $-\sqrt{29}$)	Correct distance (Allow $29/\sqrt{29}$)	A1
			(3)
(a) Way 2	$\mathbf{r}=(\mathbf{6i}+\mathbf{2j}+\mathbf{12k})+\lambda(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})$ $\therefore\ 6+3\lambda\ \ 3+\ 2-4\lambda\ \ -4\ +\ 12+2\lambda\ \ 2=5$		M1
	Substitutes the parametric coordinates of the line through (6, 2, 12) perpendicular to the plane into the cartesian equation.		
	$\lambda=-1\Rightarrow\ 3,6,10$ or $-3\mathbf{i}+4\mathbf{j}-2\mathbf{k}$	Solves for λ to obtain the required point or vector.	M1
	$\sqrt{29}$	Correct distance	A1
(a) Way 3	**Parallel plane** containing (6, 2, 12) is $\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})=34$ $\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{34}{\sqrt{29}}$	Origin to this plane is $\dfrac{34}{\sqrt{29}}$	M1
	$\Rightarrow\dfrac{\mathbf{r}.(3\mathbf{i}-4\mathbf{j}+2\mathbf{k})}{\sqrt{29}}=\dfrac{5}{\sqrt{29}}$	Origin to plane is $\dfrac{5}{\sqrt{29}}$	M1
	$\dfrac{34}{\sqrt{29}}-\dfrac{5}{\sqrt{29}}=\sqrt{29}$	Correct distance	A1
(b) **For a cross product, if the method is unclear, 2 out of 3 components should be correct for M1**	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1 & 5\\ 1 & -1 & -2\end{vmatrix}=\begin{pmatrix}3\\9\\-3\end{pmatrix}$	M1: Attempts $(2\mathbf{i}+1\mathbf{j}+5\mathbf{k})\times(\mathbf{i}-\mathbf{j}-2\mathbf{k})$ A1: Any multiple of **i + 3j – k**	M1A1
	$(\cos\theta)=\dfrac{(\mathbf{3i}\text{-}\mathbf{4j}+\mathbf{2k}).(\mathbf{i}+\mathbf{3j}\text{-}\mathbf{k})}{\sqrt{3^2+4^2+2^2}\sqrt{1^2+3^2+1^2}}\quad\left(=\dfrac{-11}{\sqrt{29}\sqrt{11}}\right)$		M1
	Attempts scalar product of normal vectors including magnitudes		
	52	Obtains angle using arccos (dependent on previous M1)	dM1 A1
	Do not isw and mark the final answer e.g. 90 – 52 = 38 loses the A1		(5)
(c)	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1 & 3 & -1\\ 3 & -4 & 2\end{vmatrix}=\begin{pmatrix}2\\-5\\-13\end{pmatrix}$	M1: Attempt cross product of normal vectors A1: Correct vector	M1A1
	$x=0:(0,\ \dfrac{5}{2},\ \dfrac{15}{2}),\quad y=0:(1,\ 0,\ 1),\quad z=0:(\dfrac{15}{13},\ \dfrac{-5}{13},\ 0)$		M1A1
	M1: Valid attempt at a point on both planes. A1: Correct coordinates **May use way 3 to find a point on the line**		
	$\mathbf{r}\times(\text{-}\mathbf{2i}+\mathbf{5j}+\mathbf{13k})=-\mathbf{5i}\text{-}\mathbf{15j}+\mathbf{5k}$	M1: $\mathbf{r}\times$ dir = pos.vector $\times$ dir **(This way round)** A1: Correct equation	M1A1
			(6)

Question Number	Scheme		Marks
(c) Way 2	"$x + 3y - z = 0$" and $3x - 4y + 2z = 5$ uses their cartesian form of and eliminate x, or y or z and substitutes back to obtain two of the variables in terms of the third		M1
	$(x = 1 - \frac{2}{5}y$ and $z = 1 + \frac{13}{5}y)$ or $(y = \frac{5z-5}{13}$ and $x = \frac{15-2z}{13})$ or $(y = \frac{5-5x}{2}$ and $z = \frac{15-13x}{2})$		A1
	Cartesian Equations: $x = \frac{y - \frac{5}{2}}{-\frac{5}{2}} = \frac{z - \frac{15}{2}}{-\frac{13}{2}}$ or $\frac{x-1}{-\frac{2}{5}} = y = \frac{z-1}{\frac{13}{5}}$ or $\frac{x - \frac{15}{13}}{-\frac{2}{13}} = \frac{y + \frac{5}{13}}{\frac{5}{13}} = z$		
	Points and Directions: Direction can be any multiple $(0, \frac{5}{2}, \frac{15}{2}), \mathbf{i} - \frac{5}{2}\mathbf{j} - \frac{13}{2}\mathbf{k}$ or $(1,0,1), -\frac{2}{5}\mathbf{i} + \mathbf{j} + \frac{13}{5}\mathbf{k}$ or $(\frac{15}{13}, -\frac{5}{13}, 0), -\frac{2}{13}\mathbf{i} + \frac{5}{13}\mathbf{j} + \mathbf{k}$		M1 A1
	M1:Uses their Cartesian equations correctly to obtain a point and direction A1: Correct point and direction – it may not be clear which is which – i.e. look for the correct numbers either as points or vectors		
	Equation of line in required form: e.g. $\mathbf{r} \times (\mathbf{-2i + 5j + 13k}) = \mathbf{-5i - 15j + 5k}$ Or Equivalent		M1 A1
			(6)
			Total 14
(c) Way 3	$\begin{pmatrix} 2\lambda + \mu \\ \lambda - \mu \\ 5\lambda - 2\mu \end{pmatrix} . \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} = 5 \Rightarrow 12\lambda + 3\mu = 5$	M1: Substitutes parametric form of $\prod_2$ into the vector equation of $\prod_1$ A1: Correct equation	M1A1
	$\mu = \frac{5}{3}, \lambda = 0$ gives $(\frac{5}{3}, -\frac{5}{3}, \frac{10}{3})$ $\mu = 0, \lambda = \frac{5}{12}$ gives $(\frac{5}{6}, \frac{5}{12}, \frac{25}{12})$ Direction $\begin{pmatrix} -2 \\ 5 \\ 13 \end{pmatrix}$	M1: Finds 2 points and direction A1: Correct coordinates and direction	M1A1
	Equation of line in required form: e.g. $\mathbf{r} \times (\mathbf{-2i + 5j + 13k}) = \mathbf{-5i - 15j + 5k}$ Or Equivalent		M1A1
	Do not allow 'mixed' methods – mark the best single attempt		
	NB for checking, a general point on the line will be of the form: (1 - 2λ, 5λ, 1 + 13λ)		

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